JAN 2006 C1

1. Factorise $x^3 - 4x^2 + 3x$.

Think: no constant, so x is a factor

$x^3 - 4x^2 + 3x$

$= x(x^2 - 4x + 3)$

$= x(x-3)(x-1)$

Integer factors of 3 are 1×3 or -1×-3, and $\underline{-1-3=-4}$

Please note! Your answers should look like this – do not leave out your working, or write answers without an = to show what they are

2. $u_{n+1} = (u_n - 3)^2$, $u_1 = 1$

(a) $u_2 = (u_1 - 3)^2 = (1-3)^2 = (-2)^2 = 4$

$u_3 = (4-3)^2 = 1$

$u_4 = (1-3)^2 = 4$

(b) Can see that ~~ddd~~ $u_n = 4$ for all even n

$\therefore u_{20} = 4$

3. L is $y = 5 - 2x$.

(a) at $x = 3$, y on L will be $5 - 2 \times 3 = 5 - 6 = -1$

$\therefore (3, -1)$ is on line L.

(b) Gradient of L is -2, gradient of normal

$= \frac{-1}{-2} = 1/2$.

3 (b)+ $y - y_1 = m(x - x_1)$

$y - (-1) = \frac{1}{2}(x - 3)$

$= y + 1$

$2y + 2 = x - 3$

$x - 2y - 5 = 0$

4. $y = 2x^2 - \frac{6}{x^3}, \quad x \neq 0$

$= 2x^2 - 6x^{-3}$

(a) $dy/dx = 2(2x) - 6(-3x^{-4})$

$= 4x + 18x^{-4}$

(b) $\int y\,dx = \int 2x^2 - 6x^{-3}\,dx = 2\left(\frac{x^3}{3}\right) - 6\left(\frac{x^{-2}}{-2}\right) + c$

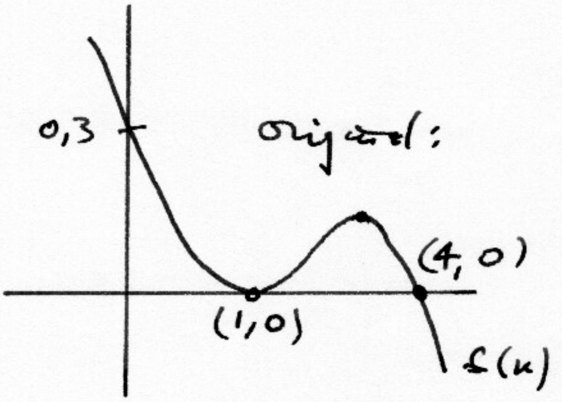
$= \frac{2}{3}x^3 + 3x^{-2} + c$

5. (a) $\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

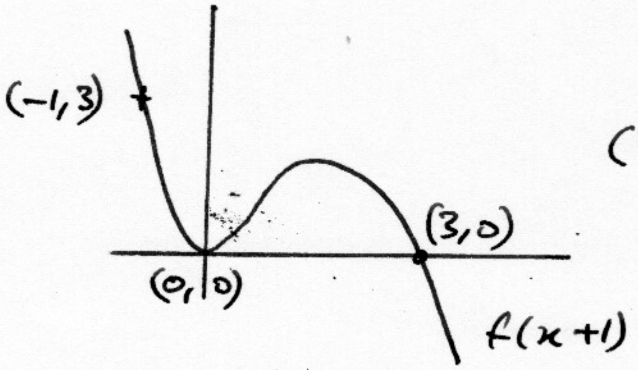
(b) $\frac{2(3+\sqrt{5})}{(3-\sqrt{5})} \cdot \frac{(3+\sqrt{5})}{(3+\sqrt{5})} = \frac{2(9 + 6\sqrt{5} + 5)}{9-5} = \frac{2}{4}(14 + 6\sqrt{5})$

$= 7 + 3\sqrt{5}$

6.

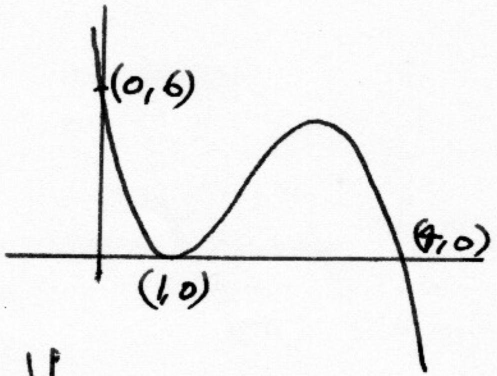


(a)



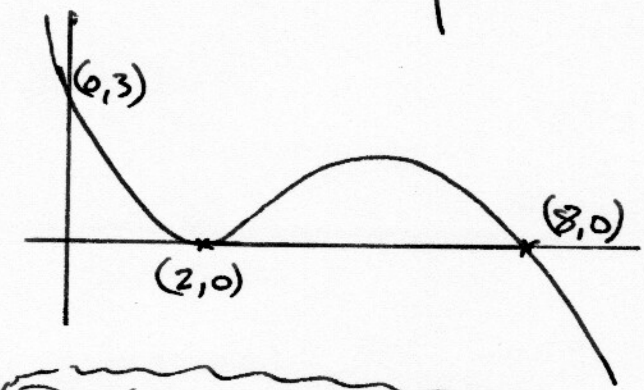
(Translated −1 in +x direction)

(b)



(stretched ×2 in y-direction).

(c)



(stretched ×2 in x-direction).

Think: At $x = 2$,
$f(\tfrac{1}{2}x) = f(\tfrac{1}{2} \times 2) = f(1) = 0$,
ditto at $x = 8$.

7. 11^{th} birthday : £500

12^{th} birthday : £700

Series is $500 + 700 + 900 + \dots$,

$a = £500$, $d = £200$

If $n = 1$ for the first term, $n = \text{age} - 10$.

(a) $£500 + £700 = £1200$

(b) $u_n = a + (n-1)d$, age $= 18$, $n = 8$,

$u_8 = 500 + 7 \times 200 = £1900$

(or simply count £500 + seven increments).

(c) $S_n = \frac{n}{2}(a + l)$

$S_8 = \frac{8}{2}(500 + 1900) = 4 \times 2400 = £9600$

(d) $S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(2 \times 500 + (n-1)200) = 32000$

$\div 100$ $\frac{n}{2}(10 + 2n - 2) = \frac{n}{2}(8 + 2n) = n(4 + n) = 320$

$n^2 + 4n - 320 = 0$

Factors of -320 (need 1 positive & 1 negative) that add to $+4$ are 20×-16

$(n + 20)(n - 16) = 0$, $n = -20$ or $+16$

Negative n not sensible, so $n = 16$, age $= 16 + 10 = \underline{26}$

8. $f'(x) = 3 + \dfrac{5x^2+2}{x^{1/2}} \qquad (x>0)$

$= 3 + 5x^{3/2} + 2x^{-1/2}$

$f(x) = \int f'(x)\,dx = 3x + 5\left(\dfrac{x^{5/2}}{(5/2)}\right) + 2\left(\dfrac{x^{1/2}}{(1/2)}\right) + c$

$= 3x + 5\left(\frac{2}{5}\right)x^{5/2} + 4x^{1/2} + c$

$= 3x + 2x^{5/2} + 4x^{1/2} + c$

At $(1,6)$, $f(1) = 3+2+4+c = 9+c = 6$

$\therefore c = 6-9 = -3$

$f(x) = 3x + 2x^{5/2} + 4x^{1/2} - 3$

9. $y = (x-1)(x^2-4)$

(a) At $y=0$, $(x-1)(x^2-4) = 0$ so $x-1=0 \Rightarrow x=1$

or $x^2-4=0$, $x^2=4$,

$x = \pm 2$

x-coordinates: P, $x=-2$

Q, $x=+2$.

(b) $y = (x-1)(x^2-4) = x^3 - x^2 - 4x + 4$

$\therefore dy/dx = 3x^2 - 2x - 4$ (using $d/dx(x^n) = nx^{n-1}$).

(c) At $(-1,6)$, $dy/dx = 3(-1)^2 - 2(-1) - 4 = 3+2-4 = 1$

= gradient of tangent.

$y - 6 = 1(x-(-1)) = x+1$, $y = x+7$

(d) To find all points where $dy/dx = 1$, set $3x^2-2x-4=1$,

$3x^2-2x-5=0$. $\quad 3\times -5 = -15$, $3-5=-2$

9(d) +

$$3x^2 - 2x - 5 = 3x^2 + 3x - 5x - 5$$

$$= 3x(x+1) - 5(x+1)$$

$$= (3x-5)(x+1) = 0$$

[or write as $\left(3x \quad \frac{-5}{1}\right)\left(1x \quad \frac{+3}{3}\right) = (3x-5)(x+1)$]

$\therefore$ $x = -1$ or $5/3 = 1\tfrac{2}{3}$.

At $x = 5/3$, $y = (x-1)(x^2-4) = 2/3\left(\frac{25}{9} - \frac{36}{9}\right)$

$$= 2/3 \times \frac{-11}{9} = \frac{-22}{27}$$

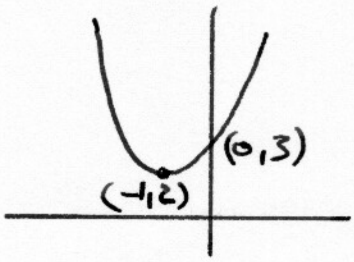
R is at $\left(1\tfrac{2}{3}, \frac{-22}{27}\right)$.

10.

(a) $\underbrace{x^2 + 2x}_{\text{(equivalent)}} + 3 = \left[\underbrace{(x+1)^2 - 1^2}_{\text{(equivalent)}}\right] + 3 = (x+1)^2 + 2$,

$a = 1$, $b = 2$.

(b) Minimum at $(-1, 2)$ (where $(x+1)^2 = 0^2 = 0$)

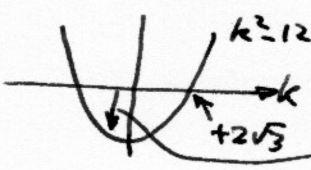


(c) For $ax^2 + bx + c = 0$, discriminant is $b^2 - 4ac$.

Here $a=1$, $b=2$, $c=3$, $b^2 - 4ac = 4 - 4\times3 = 4 - 12 = -8$

$\Rightarrow$ no real roots, does not touch or cut the x-axis.

(d) $x^2 + kx + 3 = 0$, discriminant $b^2 - 4ac = k^2 - 12 < 0$ if no real roots. Critical values at $k^2 - 12 = 0$, $k^2 = 12$, $k = \pm 2\sqrt{3}$



$k^2 - 12 < 0$ here $\Rightarrow$ need $-2\sqrt{3} < k < 2\sqrt{3}$.